Sensor Fusion and Optimal Platform Trajectory Planning for Ground Target Localization with Terrain Uncertainty and Measurement Biases

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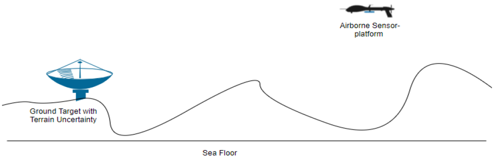
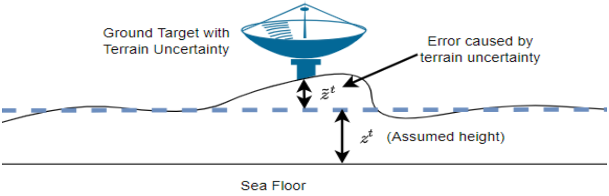
Motivation



- 3-D ground target localization with biased angle-only sensor.
- Two key sources of uncertainties:
  - Measurement bias, i.e. bias in the elevation and bearing angles.
  - Terrain uncertainty.
- Ground target can be:
  - Stationary.
  - Moving with a nearly constant velocity (CV).

Terrain Uncertainty

- How to make the 3-D localization problem observable?
  - The target height from the sea level is available.
  - Height information is obtained from a Digital Terrain Elevation Database (DTED).
- Height information obtained from DTED has uncertainty.
- Neglecting terrain uncertainty leads to:
  - Localization error.
  - Optimistic performance bounds.



Localization using Angle-only Sensor [1]

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- Tracking a ground target in 3-D using angle-only airborne sensor.
- Ground target height from the sea level is assumed to be known with terrain uncertainty.
  - Sources of uncertainties: terrain uncertainty and measurement bias.
- Bias estimation using a target of opportunity.
  - Optimal number of target of opportunity.
  - Optimal sensor trajectory.
- Target localization using bias compensated non-linear filtering.
- Evaluation of biased posterior Cramer Rao Lower Bound (PCRLB) and estimator performance evaluation.

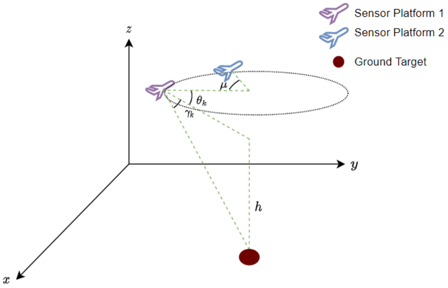
[1] D. Mitra, A. Balachandran, R. Tharmarasa, "Ground Target Tracking Using an Airborne Angle-Only Sensor with Terrain Uncertainty and Sensor Biases," Sensors, 22, no. 2:509, January 2022, pp. 1-26.

Proposed Modifications

- Challenges:
  - Presence of measurement bias causes delay in error convergence.
  - Estimation accuracy is low.
- Proposed solution:
  - Range sensor addition to improve localization accuracy.
    - Error convergence is attained faster.
  - Platform trajectory optimization for estimation.
  - Ground target localization while handling biases.
    - Bias compensation.
    - Bias estimation.

Problem Overview

- Localizing a ground target using two airborne sensor platforms.
  - Ground target has terrain uncertainty.
  - Platform 1 contains biased angle-only sensors.
  - Platform 2 contains unbiased range sensor.
- Optimal platform trajectory based on performance bound.



Measurement Model (for Platform 1)

- Biased angle-only measurements $\theta_k \in [-\pi, \pi]$ and $\gamma_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.
- From 3-D geometry,

$$\theta_k^{\text{true}} = \tan^{-1}\left(y_k^t - y_k^{p1}, x_k^t - x_k^{p1}\right) \quad (1)$$

$$\gamma_k^{\text{true}} = \tan^{-1}\left(\sqrt{\left(x_k^t - x_k^{p1}\right)^2 + \left(y_k^t - y_k^{p1}\right)^2}, z_k^t - z_k^{p1}\right) \quad (2)$$

  - True height of the ground target from the sea level is z_k^t.
- Assumed height containing terrain uncertainty $z_g = z_1^t + \tilde{z}^t$.
  - Error associated to terrain uncertainty $\tilde{z}^t \sim \mathcal{N}(\tilde{z}^t; 0, \sigma_{z^t}^2)$.
- Measurement model for platform 1,

$$\mathbf{z}_k = \mathbf{h}_a(\mathbf{x}_k^t, \mathbf{x}_k^{p1}) + \mathbf{b}_k + \mathbf{w}_k \quad (3)$$

  - Bias vector $\mathbf{b}_k = [\theta_{b_k}, \gamma_{b_k}]$.
  - $\mathbf{h}_a(\mathbf{x}_k^t, \mathbf{x}_k^{p1}) = [\theta_k^{\text{true}}, \gamma_k^{\text{true}}]^T$.
  - Measurement noise $\mathbf{w}_k \sim \mathcal{N}(\mathbf{w}_k; 0, \mathbf{R}_k)$, where $\mathbf{R}_k$ = diag$(\sigma_\theta^2, \sigma_\gamma^2)$.

Measurement Model (for Platform 2)

- Unbiased range measurements,

$$r_k = \sqrt{(x_k^t - x_k^{p2})^2 + (y_k^t - y_k^{p2})^2 + (z_k^t - z_k^{p2})^2} \quad (4)$$

- Measurement model for platform 2,

$$\mathbf{z}_k^r = \mathbf{h}_r(\mathbf{x}_k^t, \mathbf{x}_k^{p2}) + \mathbf{w}_k^r \quad (5)$$

  - $\mathbf{h}_r(\mathbf{x}_k^t, \mathbf{x}_k^{p2}) = [r_k]$.
- Measurement noise $\mathbf{w}_k^r$ is a zero-mean Gaussian with variance $\sigma_{r_k}^2$.
- Range measurements might not be available for all time-steps.

Bias Compensated Localization

- Bias compensated measurements

$$\mathbf{z}_k^c = \mathbf{h}_a(\mathbf{x}_k^t, \mathbf{x}_k^{p1}) + \mathbf{b}_k - \mathbf{b}_k^{\text{prior}} + \mathbf{w}_k^c \quad (6)$$

  - Bias prior $\mathbf{b}_k^{\text{prior}} = [\theta_{b_k}^{\text{prior}}, \gamma_{b_k}^{\text{prior}}]$.
  - Measurement noise $\mathbf{w}_k^c \sim \mathcal{N}(\mathbf{w}_k^c; 0, \mathbf{R}_k^c)$.
  - Bias compensated covariance $\mathbf{R}_k^c = \text{diag}((\sigma_\theta^2 + \sigma^2_{\theta_{b_k}^{\text{prior}}}), (\sigma_\gamma^2 + \sigma^2_{\gamma_{b_k}^{\text{prior}}}))$.
- Considering terrain uncertainty (only for initialization) $\mathbf{R}_1 = \text{diag}\left(\sigma_{z^t}^2, \left(\sigma_{\theta_k}^2 + \sigma_{\theta_{b_1}}^2\right), \left(\sigma_{\gamma_k}^2 + \sigma_{\gamma_{b_1}}^2\right)\right)$
- Unscented Kalman Filter (UKF) is used for non-linear filtering with zero point initialization.

Joint Estimation of Target and Bias States

- No bias compensation is performed.
- Bias states are stacked with the target state.
  - Joint state initialization as $\mathbf{x}_1 = [x_1, 0, y_1, 0, z_g, \theta_{b_k}^{\text{prior}}, \gamma_{b_k}^{\text{prior}}]$.
  - For stationary target $\mathbf{x}_1 = [x_1, y_1, z_g, \theta_{b_k}^{\text{prior}}, \gamma_{b_k}^{\text{prior}}]$.
  - $\theta_{b_k}^{\text{prior}}$ and $\gamma_{b_k}^{\text{prior}}$ can be zero.
- UKF is used for non-linear filtering.
- Information on terrain uncertainty not used after initialization.

Performance Bound

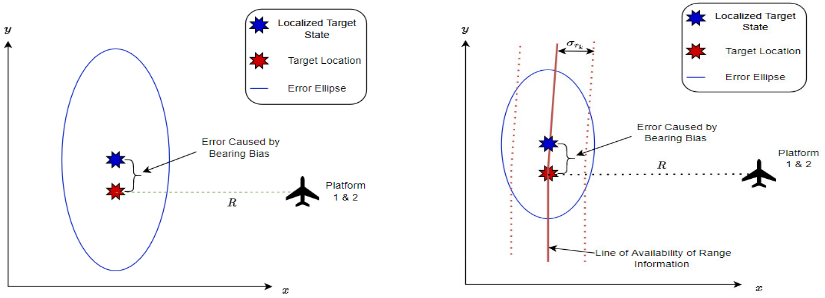
- Posterior Cramer Rao Lower Bound (PCRLB) is used.
- With $\mathbf{Z}_K = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_K]$,

$$\mathbf{C}_k = \mathbb{E}\Big[(\hat{\mathbf{x}}_k^t(\mathbf{z}_k) - \mathbf{x}_k^t)(\hat{\mathbf{x}}_k^t(\mathbf{z}_k) - \mathbf{x}_k^t)^T\Big] \geq \mathbf{J}_k^{-1} \tag{7}$$

- Fisher Information Matrix (FIM) or $\mathbf{J}_k$,

$$\mathbf{J}_{k+1} = \Big(\mathbf{F}_k \mathbf{J}_k^{-1} \mathbf{F}_k^T + \mathbf{Q}_k\Big)^{-1} + \mathbf{J}_z(k+1) \tag{8}$$

  - Measurement contribution $\mathbf{J}_z(k) = \mathbb{E}\Big[q_k \mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k\Big]$.
  - For angle-only sensors $\mathbf{H}_k = \frac{\partial \mathbf{h}_a(\mathbf{x}_k^t, \mathbf{x}_k^{p1})}{\mathbf{x}_k^t}$.
  - For range-only sensor $\mathbf{H}_k = \frac{\partial \mathbf{h}_r(\mathbf{x}_k^t, \mathbf{x}_k^{p2})}{\mathbf{x}_k^t}$
- For range and angle measurements $\mathbf{J}_z(k) = \mathbf{J}_z^a(k) + \mathbf{J}_z^r(k)$.
  - $\mathbf{J}_z(k) = \mathbf{J}_z^a(k)$ or $\mathbf{J}_z(k) = \mathbf{J}_z^r(k)$, based on the availability of the sensor.

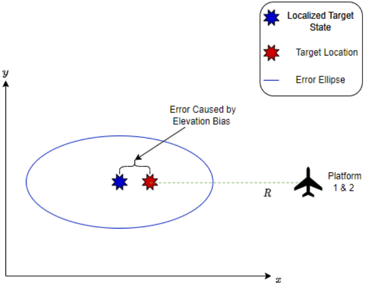


(a) Localization affected by θ_{b_k}

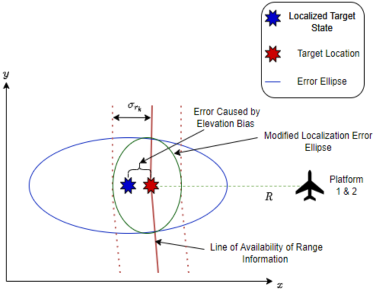
(b) Range sensor fusion with θ_{b_k}

Figure: Target state and the localization estimates in presence of biases in the angle measurements with $\mu = 0$.

Range Sensor Fusion (2)



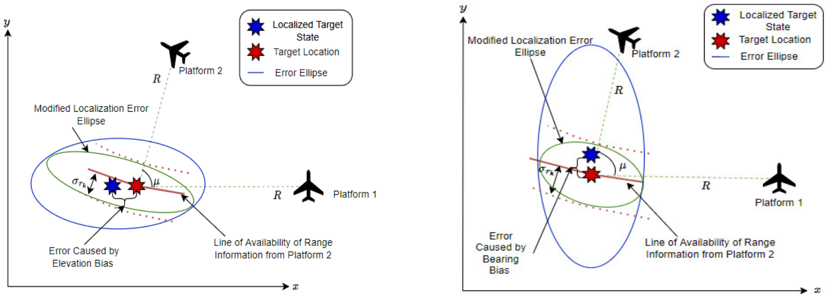
(a) Localization affected by γ_{b_k}



(b) Range sensor fusion with γ_{b_k}

Figure: Target state and the localization estimates in presence of biases in the angle measurements with $\mu = 0$.

Optimal Trajectory Planning (1)



(a) Range sensor fusion with γ_{b_k}

(b) Range sensor fusion with θ_{b_k}

Figure: x-y plane projection of stationary target state and the localization estimates with sensor fusion and $\mu > 0$.

Optimal Trajectory Planning (1)

- Goal: Finding optimal μ by optimizing PCRLB.
- Measurement contribution of the FIM,

$$\mathbf{J}_z(k) = \mathbf{J}_z^a(k) + \mathbf{J}_z^r(k) \tag{9}$$

  - Sampling rate of the range sensor might be different from the angle-only sensors.
  - $\mathbf{J}_z^r(k)$ might not be available for all the time-steps.
- The optimization problem is formulated as,

$$\begin{aligned} &\arg\min_{\mu} \sum_{k=1}^{K} \mathrm{Tr}[\mathbf{J}_k^{-1}] \\ &\text{s.t.} \quad \mu \in [0, 360^\circ) \end{aligned} \tag{10}$$

Key Takeaways

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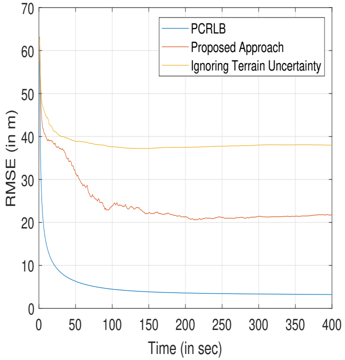
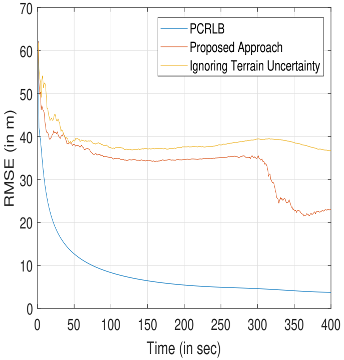
- The bias in the angle-only measurements are smaller $\longrightarrow$ Choose $\mu = 0$.
- For larger bias in the angle-only measurements $\longrightarrow$ Increase separation between the platforms.

Simulation Results (1)



(a) Stationary ground target



(b) Moving ground target

Figure: Comparison of the RMSE of the proposed approach with an approach that ignores the terrain uncertainty, when the $\sigma_{z^t} = 30$ m.

Simulation Results (2)

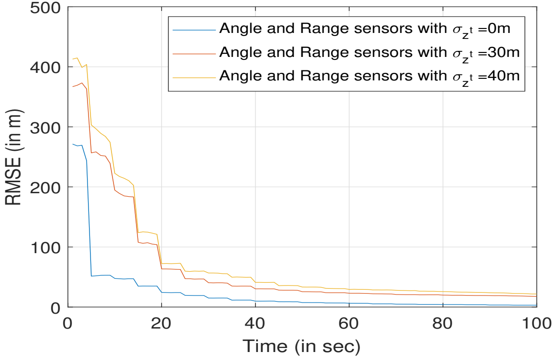


Figure: Ground target localization errors for various terrain uncertainties using range sensor fusion for the platform separation angle $\mu = 0^\circ$.

- Higher terrain uncertainty leads to higher localization error.

Simulation Results (3)

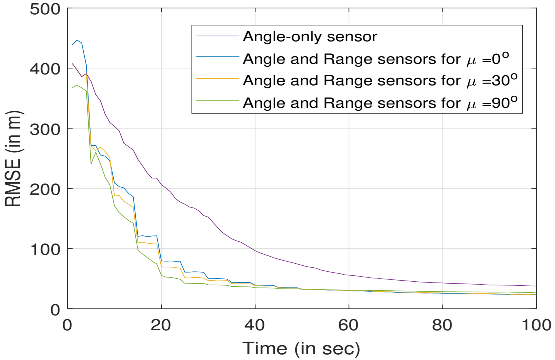
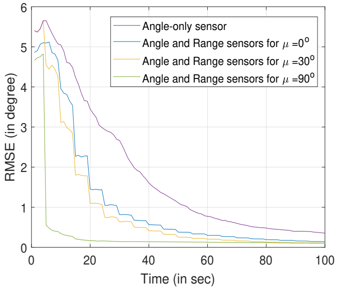
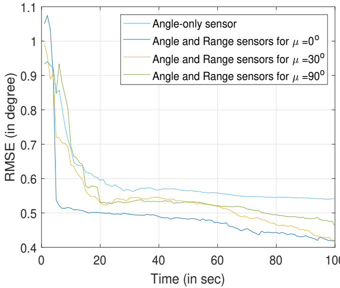


Figure: Comparison of errors in target state localization using range sensor fusion for various degrees of separations between airborne platforms.

- $\mu = 90^\circ$ leads to faster reduction in RMSE for localization.



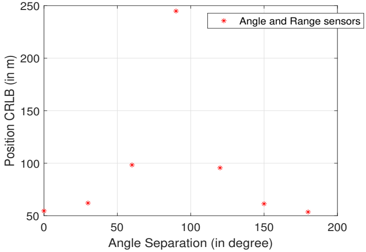
(a) Bearing bias estimation errors



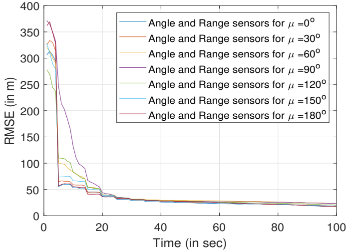
(b) Elevation bias estimation errors

Figure: Comparison of bias estimation errors for various angles of separation.

- $\mu = 90^\circ$ is optimal for bearing bias estimation.
- $\mu = 0^\circ$ is optimal for elevation bias estimation.



(a) PCRLB comparison at time = 5 s



(b) Target localization error

Figure: Comparison of ground target localization error for various μ with $\sigma_{\theta_b} = 1^\circ$, $\sigma_{\gamma_b} = 1^\circ$ and $\sigma_{z^t} = 30$m.

- Impact of the higher μ on reducing localization error fades away for small biases.
- Terrain uncertainty primarily impacts localization accuracy $\longrightarrow$ Increasing μ has no effect in reducing RMSE.

Conclusions

- Localization of a ground target using,
  - Biased angle-only sensors.
  - Additional range sensor $\longrightarrow$ Improves error convergence.
- Uncertainties present $\longrightarrow$
  - Measurement bias uncertainty.
  - Terrain uncertainty.
- Proposed localization approaches $\longrightarrow$
  - Use bias compensated measurements with inflated initial covariance.
  - Joint estimation of the bias and the target states. (Recommended method, if time is not a constraint)
- PCRLB based optimal platform trajectory planning $\longrightarrow$
  - Choose 90° separation for higher bearing biases.
  - Choose 0° separation for lower bearing biases.

Backup Slides

System Model

- Ground target state $\mathbf{x}_k^t = [x_k^t, \dot{x}_k^t, y_k^t, \dot{y}_k^t, z_k^t]$.
- State evolution of ground target,

$$\mathbf{x}_{k+1}^t = \mathbf{F}_k \mathbf{x}_k^t + \mathbf{G_k} \mathbf{v}_k \tag{11}$$

  - Process noise $\mathbf{v}_k \sim \mathcal{N}(\mathbf{v}_k; 0, \sigma_v^2)$.
  - For a stationary ground target $\dot{x}_k^t = \dot{y}_k^t = 0$ and $\mathbf{F}_k = \mathbf{I}$.
- Platform 1 and 2 with states $\mathbf{x}_k^{p1} = [x_k^{p1}, \dot{x}_k^{p1}, y_k^{p1}, \dot{y}_k^{p1}, z_k^{p1}]$ and $\mathbf{x}_k^{p2} = [x_k^{p2}, \dot{x}_k^{p2}, y_k^{p2}, \dot{y}_k^{p2}, z_k^{p2}]$.
  - Platforms follow a 'coordinated turn model'.
  - Ground target remains at the centre of the circle.
  - Circle radius R and the constant speed v are known a-priori.
  - Turn rate $\omega = \frac{v}{R}$ rad/s.
  - Angle of separation is μ.

Simulation Parameters

Platform radius (R)	3000 m
Platform speed (v)	50 m/s
Platform height (z_1^{p1})	700 m
Standard deviation of terrain uncertainty (σ_{z^t})	30 m
Range measurement standard deviation (σ_{r_k})	5 m
Bearing measurement standard deviation (σ_{θ_k})	0.4°
Elevation measurement standard deviation (σ_{γ_k})	0.2°
Sampling time of the angle-only sensor (T_1)	1 s
Sampling time of the range sensor (T_2)	5 s
Mean bearing bias (θ_{b_k})	0°
Bearing bias standard deviation ($\sigma_{\theta_{b_k}}$)	5°
Mean elevation bias (γ_{b_k})	0°
Elevation bias standard deviation ($\sigma_{\gamma_{b_k}}$)	1°

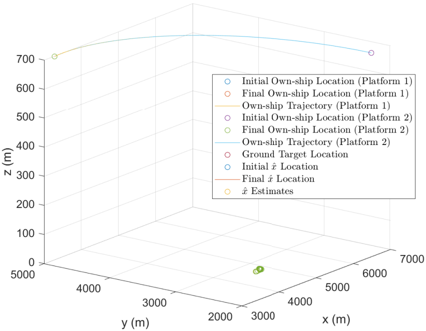


Figure: Target own-ship geometry along with the estimates for the airborne platform separation angle $\mu = 0^{\circ}$.

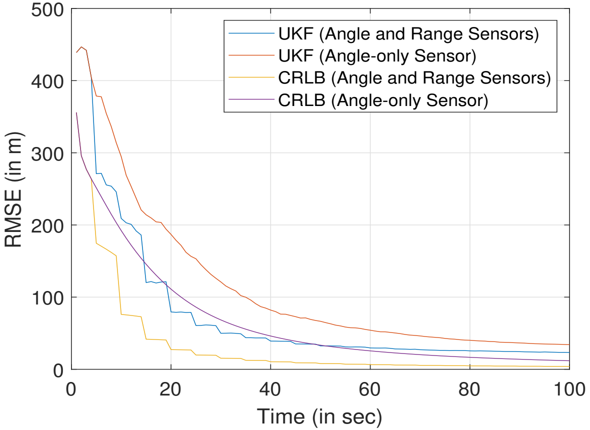
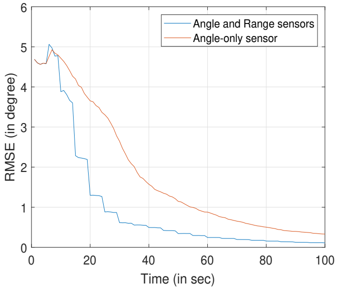
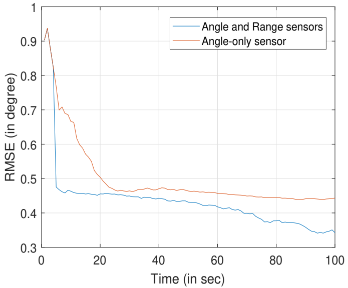


Figure: Ground target localization error using range sensor fusion for the platform separation angle $\mu = 0^\circ$ and $\sigma_{z^t} = 30$m.

Simulation Results (Backup)



(a) Bearing bias estimation



(b) Elevation bias estimation

Figure: Bias estimation errors for the airborne platform separation angle $\mu = 0^\circ$.